SLIDING IPL: AN EFFICIENT APPROACH FOR ESTIMATING THE PHASES OF LARGE SAR IMAGE TIME SERIES

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Motivation

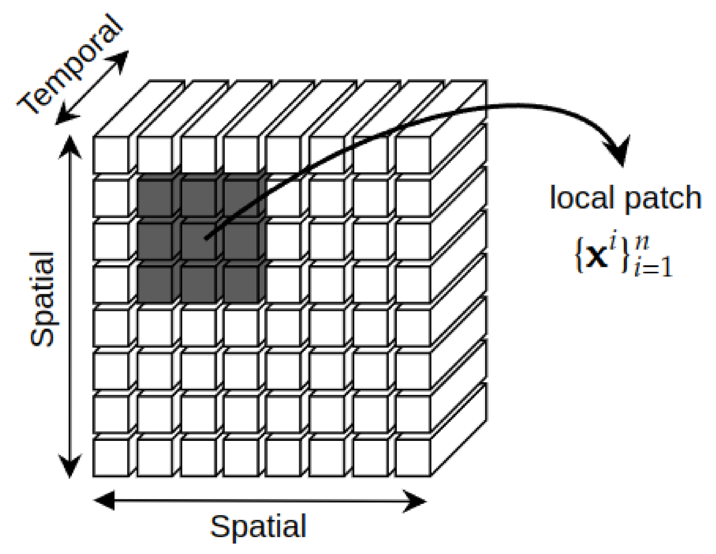
- ▲ Availability of massives **Synthetic Aperture Radar (SAR) data**
- ▲ Offline approaches require **reprocessing all data**
- ▲ Need to efficiently incorporate new acquisitions
- ▲ **Sequential processing in Interferometric SAR (InSAR)** remains underexplored

Data modeling

For a stack of l **co-registered SAR images**, we use a sliding window of n pixels representing the homogeneous neighborhood $\{\mathbf{x}^i\}_{i=1}^n$, with $\mathbf{x}^i \in \mathbb{C}^l$. Each pixel of the local patch

$$\mathbf{x}^i = [x_1^i, \dots, x_l^i]^T \in \mathbb{C}^l$$

is assumed to be distributed as a zero mean Complex Circular Gaussian (CCG) , i.e. $\mathbf{x} \sim \mathcal{CN}(0, \mathbf{\Sigma})$.



The **Covariance Matrix (CM)** for the SAR image time series is

$$\mathbf{\Sigma} = \text{mod}(\mathbf{\Sigma}) \odot \text{arg}(\mathbf{\Sigma}) = \mathbf{\Psi} \odot \mathbf{w}\mathbf{w}^H$$

where $\mathbf{\Psi}$ denotes the **coherence matrix** and $\mathbf{w}$ is the vector containing **the exponential of the complex phases**, i.e. $\mathbf{w} = [e^{j\theta_0}, \dots, e^{j\theta_l}] \in \mathbb{T}_l$, where

$$\mathbb{T}_l = \{\mathbf{w} \in \mathbb{C}^l \mid |[\mathbf{w}]_i| = 1, \forall i \in [1, l]\}$$

is the l-torus of phase only complex vector.

Interferometric Phase Linking (IPL)

IPL algorithms recover $\mathbf{w}$ from $\{x_i\}_{i=1}^n$ by fitting a **plug-in CM** $\hat{\mathbf{\Sigma}}$ using COvariance Fitting Interferometric Phase Linking (COFI-PL) [3].

$$\begin{aligned} \underset{\mathbf{w}}{\text{minimize}} \quad & f_{\hat{\mathbf{\Sigma}}}^d(\mathbf{w}) \triangleq d^2(\hat{\mathbf{\Sigma}}, \hat{\mathbf{\Psi}} \odot \mathbf{w}\mathbf{w}^H) \\ \text{s.t.} \quad & \theta_1 = 0 \\ & \mathbf{w} \in \mathbb{T}_l \end{aligned}$$

▲ **Choice of matrix distance**
Euclidean distance instead of the Kullback-Leibler divergence, as it has been shown to perform better [3].

$$\begin{aligned} f_{\hat{\mathbf{\Sigma}}}^{\text{E}}(\mathbf{w}) &= ||\hat{\mathbf{\Sigma}} - \hat{\mathbf{\Psi}} \odot \mathbf{w}\mathbf{w}^H||_2^2 \\ &= -2\mathbf{w}^H(\mathbf{\Psi} \odot \hat{\mathbf{\Sigma}})\mathbf{w}. \end{aligned}$$

▲ **Choice of covariance matrix plug-in**
Phase-Only Sample Covariance Matrix (PO) estimator, which significantly outperforms the Sample Covariance Matrix (SCM) [3].

$$\hat{\mathbf{\Sigma}}_{PO} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{y}^i \mathbf{y}^{iH}$$

where $\mathbf{y} = \Phi_{\mathbb{T}}(\mathbf{x})$ and $\Phi_{\mathbb{T}} : x = re^{i\theta} \to e^{i\theta}$. The **tapering** operator is defined as

$$[\mathbf{W}(b)]_{ij} = \begin{cases} 1 & \text{if} \quad |i - j| \leq b \\ 0 & \text{otherwise.} \end{cases}$$

The regularized estimator has, then, the following form

$$\hat{\mathbf{\Sigma}}_{BW-PO} = \mathbf{W}(b) \odot \hat{\mathbf{\Sigma}}_{PO}.$$

Sliding IPL

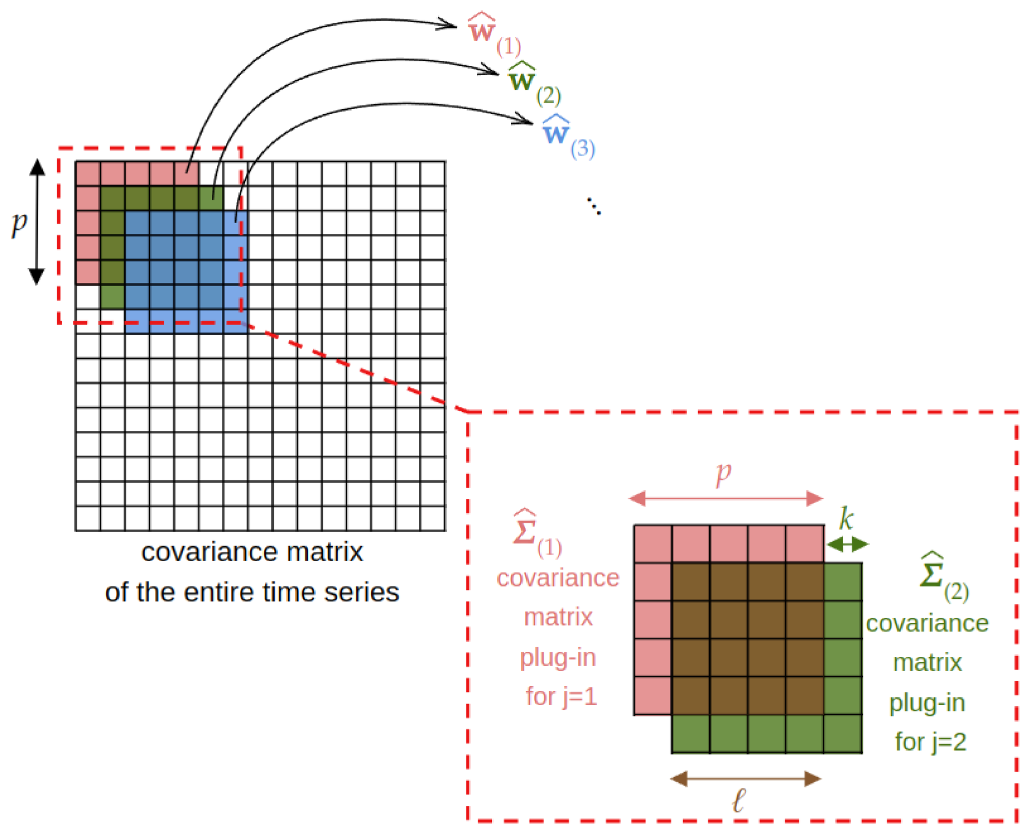
At each j:

- ▲ Slide a **temporal window** of size p with stride k (overlap $\ell = p - k$)
- ▲ **Estimate phase vector** $\hat{\mathbf{w}}_{(j)}$ and covariance matrix $\hat{\mathbf{\Sigma}}_{(j)}$
- ▲ Solve the regularized IPL problem

$$\begin{aligned} \underset{\mathbf{w}_{(j)}}{\text{minimize}} \quad & f_{\hat{\mathbf{\Sigma}}}^{\text{E}}(\mathbf{w}_{(j)}) + \lambda h(\mathbf{w}_{(j)}) \\ \text{s.t.} \quad & \mathbf{w}_{(j)} \in \mathbb{T}_p. \end{aligned}$$

where h is a penalty term that guarantees **the phase estimates to stay close to the previous ones within the window overlap**

$$h(\mathbf{w}_{(j)}) = \left\| \begin{pmatrix} \mathbf{w}_{\ell(j-1)} \\ \mathbf{0} \end{pmatrix} - \mathbf{w}_{(j)} \right\|^2$$



Majorization-Minimization (MM) algorithm steps:

- **Repeat:**
  1. Construct a surrogate function $g(\mathbf{w}_{(j)}|\mathbf{w}_{(j)}^{(t)})$ that majorizes the original objective

$$g(\mathbf{w}_{(j)}|\mathbf{w}_{(j)}^{(t)}) = -\text{Re}\left(\mathbf{w}_{(j)}^H \left[4(\hat{\mathbf{\Psi}}_{(j)} \circ \hat{\mathbf{\Sigma}}_{(j)})\mathbf{w}_{(j)}^{(t)} + 2\lambda \begin{pmatrix} \mathbf{w}_{\ell(j-1)} \\ \mathbf{0} \end{pmatrix}\right]\right)$$

  2. Minimize the surrogate

$$\mathbf{w}_{(j)} = \arg\min_{\mathbf{w}_{(j)}} g(\mathbf{w}_{(j)}|\mathbf{w}_{(j)}^{(t)}).$$

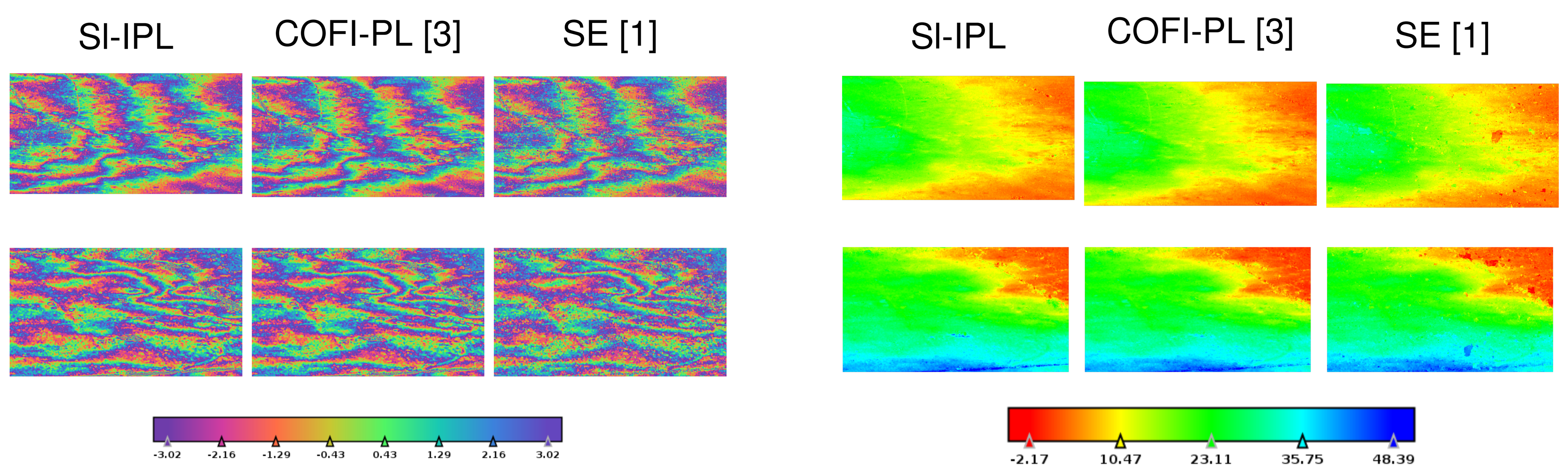
- **Until convergence.**

Output: $\mathbf{w}_{(j)}$

Real data application

Mexico City

- ▲ Time series: $l = 30$ Sentinel-1 SAR images acquired every 12 days
- ▲ Acquisition period: from $03/08/2019$ to $16/07/2020$ (11 months)
- ▲ Multilooking window size $n = 49$, with $p = 5$, $\ell = 4$, and $k = 1$



method	colinearity [2]	SSIM	time	complexity
SI-IPL	**0.90**	**0.94**	**3.9 min**	$\boldsymbol{O(p^2)}$
Sequential IPL [1]	0.84	0.85	17.1 min	$O(p^3 + i^3)$
COFI-PL [3]	**0.91**	Ref	92.4 min	$O(l^2)$

Conclusions and perspectives

We present a novel covariance-fitting-based approach for **SAR phase estimation** in IPL that efficiently **incorporates new images without storing past data**, achieves **performance comparable to COFI-PL**, **reduces computation time**, and remains flexible for any cost function and plug-in.

Perspectives

- ▲ Choose another type of area (natural environment)
- ▲ Automatic estimation of λ

References

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