Sequential Phase Linking: Progressive Integration of SAR Images for Operational Phase Estimation

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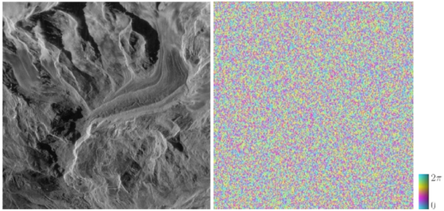
Content

Content

SAR Image

Single Look Complex (SLC) is characterized by a complex signal $z = a \cdot e^{j\theta}$

- **Amplitude (a)**: intensity of the backscattering
- **Phase (θ)**: geometric information, random information $\theta = \theta_{\text{random}} + \theta_{\text{geometric}}$



Sentinel-1 SAR image - glacier

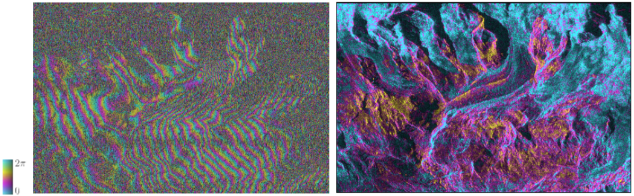
Interferometry SAR

2 coregistered SLC images of the same scene at different times → interferogram :

$$\gamma e^{j\theta}(i,j) = \frac{\sum_{i,j\in\Omega} z_1(i,j) z_2^*(i,j)}{\sqrt{\sum_{i,j\in\Omega} z_1(i,j) z_1^*(i,j) \sum_{i,j\in\Omega} z_2(i,j) z_2^*(i,j)}}$$

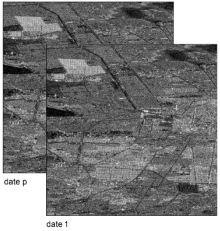
where:

- γ is **coherence** ($\in [0,1]$), representing the similarity between the two images
- θ represents the **phase difference** ($\theta = \theta_1 - \theta_2$, $\theta \in [-\pi, \pi]$)



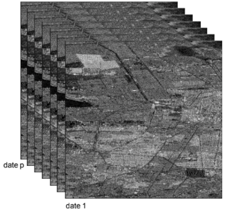
(a) phase (b) coherence

Multi-temporal InSAR



2-Pass interferometry

- Unsatisfactory results
- Measurement accuracy (centimetric)

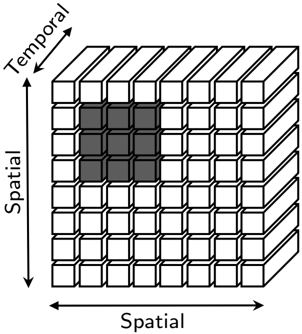


Multi-temporal interferometry

- Continuous monitoring of Earth deformations
- Improvement in measurement accuracy (millimetric)
- Building interferometric networks from a time series of SAR images

Content

Data model



Representation of SAR time series using a sliding window of n pixels $\widetilde{x}^i$ [3]

Consider a multivariate random vector

$\widetilde{\mathbf{x}} = [\widetilde{x}_1, \dots, \widetilde{x}_p]^T, \quad \forall\,(l,k) \in [0, p-1]^2$

$\{\widetilde{\mathbf{x}}^i\}_{i=1}^n \quad \forall i \in [1, n]$, a set i.i.d $\longrightarrow \widetilde{\mathbf{x}} \sim \mathcal{CN}(0, \widetilde{\mathbf{\Sigma}})$

The log-likelihood is

$$\mathcal{L}(\widetilde{\mathbf{x}}; \widetilde{\mathbf{\Sigma}}) = -\log\left(\prod_{i=1}^{n} f(\widetilde{\mathbf{x}}^i, \widetilde{\mathbf{\Sigma}})\right)$$

$$\propto n\log(|\widetilde{\mathbf{\Sigma}}|) + n\,\mathrm{Tr}(\widetilde{\mathbf{\Sigma}}^{-1}\mathbf{S})$$

where $\mathbf{S} = \frac{1}{n}\sum_{i=1}^{n} \widetilde{\mathbf{x}}^i \widetilde{\mathbf{x}}^{iH}$

Covariance matrix : $\mathbb{E}[\widetilde{\mathbf{x}}\widetilde{\mathbf{x}}^H] \triangleq \widetilde{\mathbf{\Sigma}} = \widetilde{\mathbf{\Psi}} \odot \widetilde{\mathbf{w}}_\theta \widetilde{\mathbf{w}}_\theta^H$

[3] P. Vu, A. Breloy, F. Brigui, Y. Yan, and G. Ginolhac, "A new phase linking algorithm for multi-temporal INSAR based on the maximum likelihood estimator" IGARSS International Geoscience and Remote Sensing Symposium, IEEE, 2022

Classic PL

Principle:
Estimate $p-1$ phase differences from p SAR images $\longrightarrow \widetilde{\mathbf{w}}_\theta$

Assuming that $\widetilde{\boldsymbol{\Psi}}$ is known, the method is equivalent to optimizing the following problem [2]:

$$\underset{\widetilde{\mathbf{w}}_\theta}{\text{minimize}} \quad \widetilde{\mathbf{w}}_\theta^H (\widetilde{\boldsymbol{\Psi}}^{-1} \circ \mathbf{S}) \widetilde{\mathbf{w}}_\theta$$
$$\text{subject to} \quad \theta_1 = 0$$

PROBLEM !!!
In reality, $\widetilde{\boldsymbol{\Psi}}$ is unknown

- [2] proposed to use a plug-in: $\widetilde{\boldsymbol{\Psi}}_{mod} = |\mathbf{S}| \longrightarrow$ **not optimal**
- [3] proposed to estimate $\widetilde{\boldsymbol{\Psi}}$ jointly with $\widetilde{\mathbf{w}}_\theta$

[2] Guarnieri, Andrea Monti, and Stefano Tebaldini. "On the exploitation of target statistics for SAR interferometry applications." IEEE Transactions on Geoscience and Remote Sensing 46.11 (2008): 3436-3443.

[3] P. Vu, A. Breloy, F. Brigui, Y. Yan, and G. Ginolhac, "A new phase linking algorithm for multi-temporal INSAR based on the maximum likelihood estimator", IGARSS International Geoscience and Remote Sensing Symposium, IEEE, 2022

Phase Linking based on Maximum Likelihood Estimation (MLE-PL)

→ The optimization problem [3]:

$$\underset{\widetilde{\boldsymbol{\Psi}}, \widetilde{\mathbf{w}}_\theta}{\text{minimize}} \quad \mathcal{L}_{\mathcal{G}}(\widetilde{\mathbf{x}}^i; \widetilde{\boldsymbol{\Sigma}}(\widetilde{\boldsymbol{\Psi}}, \widetilde{\mathbf{w}}_\theta))$$

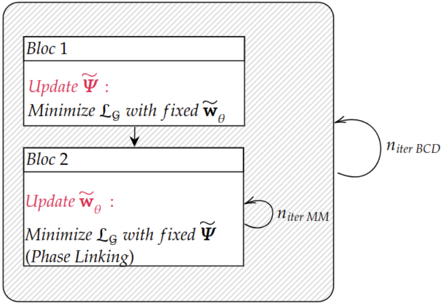
$$= n \log(|\widetilde{\boldsymbol{\Sigma}}|) + \sum_{i=1}^{n} \widetilde{\mathbf{x}}^{iH} \widetilde{\boldsymbol{\Sigma}}^{-1} \widetilde{\mathbf{x}}^i$$

subject to $\theta_1 = 0$

$\widetilde{\mathbf{w}}_\theta \in \mathbb{T}_p$

$\widetilde{\boldsymbol{\Psi}}$ real symmetric

with $\mathbb{T}_p = \{\widetilde{\mathbf{w}} \in \mathbb{C}^p \| |[\widetilde{w}]_i| = 1, \forall i \in [1, p]\}$



2 unknowns to estimate ⟶ **Block Coordinate Descent Algorithm (BCD)**

[3] P. Vu, A. Breloy, F. Brigui, Y. Yan, and G. Ginolhac. "A new phase linking algorithm for multi-temporal INSAR based on the maximum likelihood estimator." IGARSS International Geoscience and Remote Sensing Symposium. IEEE, 2022.

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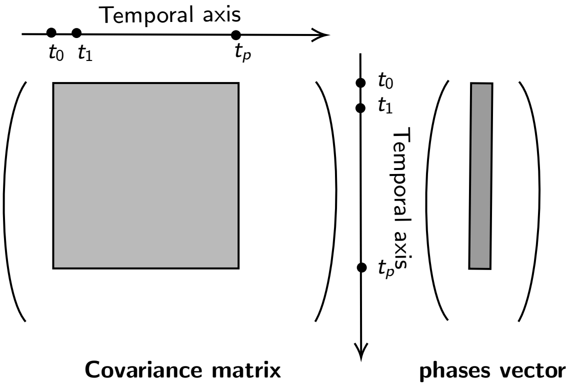
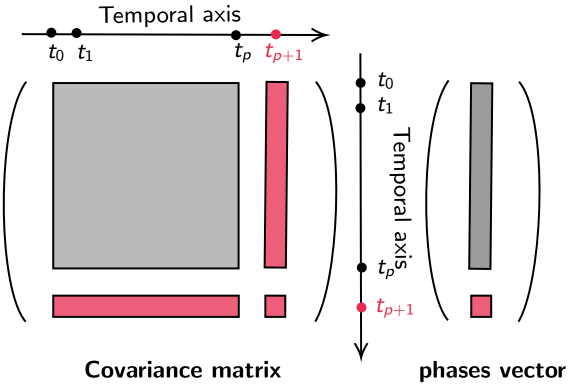
A new image arrives



$\forall i = 1, \ldots, n$

$$\tilde{\mathbf{x}}^i = \begin{pmatrix} x_1^i \\ x_2^i \\ \vdots \\ x_p^i \end{pmatrix}_{(p,1)}$$

A new image arrives



$\forall i = 1, \ldots, n$

$$\tilde{\mathbf{x}}^i = \begin{pmatrix} x_1^i \\ x_2^i \\ \vdots \\ x_p^i \end{pmatrix}_{(p,1)} \rightarrow \tilde{\mathbf{x}}^i = \begin{pmatrix} x_1^i \\ x_2^i \\ \vdots \\ x_p^i \\ \mathbf{x}_{p+1}^i \end{pmatrix}_{(p+1,1)}$$

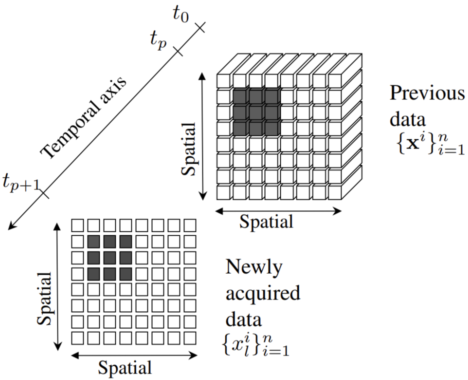
At each new SAR acquisition,

- Re-Estimation of the increasing covariance matrix
- Re-Estimation of the phases

$\longrightarrow$ **Huge computation time**

Problem : Development of a new and sequential multi-temporal interferometry SAR approach for estimating SAR phase time series using statistical tools.

Data model



Representation of SAR time series with a sliding window containing n pixels $\tilde{\mathbf{x}}^i$

We consider a set $\{\tilde{\mathbf{x}}^i\}_{i=1}^n$ where

$$\tilde{\mathbf{x}}^i = [\underbrace{x_1^i, \ldots, x_p^i}_{\mathbf{x}^i}, x_l^i]^T \in \mathbb{C}^{l=p+1}$$

$\{\tilde{\mathbf{x}}^i\}_{i=1}^n \ \forall i \in [1, n]$, a set i.i.d $\longrightarrow \tilde{\mathbf{x}} \sim \mathcal{CN}(0, \tilde{\boldsymbol{\Sigma}})$

The covariance matrix can be rewritten as

$$\tilde{\boldsymbol{\Sigma}} = \begin{pmatrix} \Sigma & w_{\theta_l}^* \mathrm{diag}(\mathbf{w}_\theta)\gamma^T \\ \gamma \mathrm{diag}(\mathbf{w}_\theta)^H w_{\theta_l} & \gamma_l \end{pmatrix}$$

Maximum Likelihood Estimation (MLE) problem

$$\underset{\tilde{\boldsymbol{\Psi}}, \tilde{\mathbf{w}}_\theta}{\text{minimize}} \quad \mathcal{L}_{\mathcal{G}}(\tilde{\mathbf{x}}^i; \tilde{\boldsymbol{\Sigma}}(\tilde{\boldsymbol{\Psi}}, \tilde{\mathbf{w}}_\theta))$$
$$\text{subject to} \quad \theta_1 = 0, \tilde{\mathbf{w}}_\theta \in \mathbb{T}_p, \tilde{\boldsymbol{\Psi}} \text{ real symmetric}$$

$$\underset{\gamma, \gamma_l, w_{\theta_l}}{\text{minimize}} \quad \mathcal{L}_{\mathcal{G}}(\tilde{\mathbf{x}}^i; \gamma, \gamma_l, w_{\theta_l})$$
$$\text{subject to} \quad \gamma, \gamma_l \text{ real}, |w_{\theta_l}| = 1, \theta_1 = 0$$

w_{θ_l} is estimated with :

- estimated past
  - $\hat{\mathbf{w}}_\theta$
  - $\hat{\boldsymbol{\Sigma}}$
- new data

statistics of the conditional distribution of new image with respect to the past

$$\mathcal{L}_{\mathcal{G}}(\tilde{\mathbf{x}}^i; \gamma, \gamma_l, w_{\theta_l}) = -\sum_{i=1}^{n} \mathcal{L}_{\mathcal{G}}{}^i(x_l^i | \mathbf{x}^i; \gamma, \gamma_l, w_{\theta_l}) + \mathcal{L}_{\mathcal{G}}{}^i(\mathbf{x}^i)$$

According to [1], $x_l^i | \mathbf{x}^i \sim \mathcal{CN}(\mu_x^i, \sigma_x^2)$ where

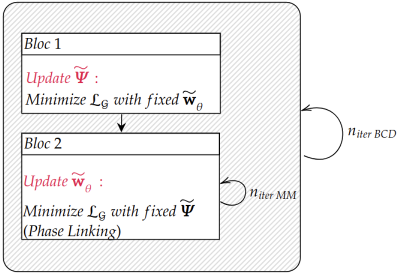
$$\mu_x^i = w_{\theta_l} \gamma \text{diag}(\hat{\mathbf{w}}_\theta)^H \hat{\boldsymbol{\Sigma}}^{-1} \mathbf{x}^i,$$
$$\sigma_x^2 = \gamma_l - \gamma \text{diag}(\hat{\mathbf{w}}_\theta)^H \hat{\boldsymbol{\Sigma}}^{-1} \text{diag}(\hat{\mathbf{w}}_\theta^H) \gamma^T \times$$

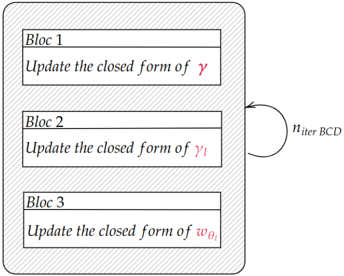
[1] T. W. Anderson, An introduction to multivariate statistic alanalysis, vol. 2, Wiley New York, 1958.

MLE-PL vs S-MLE-PL

$$\underset{\tilde{\boldsymbol{\Psi}}, \tilde{\mathbf{w}}_\theta}{\text{minimize}} \quad \mathcal{L}_{\mathcal{G}}(\tilde{\mathbf{x}}^i; \tilde{\boldsymbol{\Sigma}}(\tilde{\boldsymbol{\Psi}}, \tilde{\mathbf{w}}_\theta))$$
$$\theta_1 = 0, \tilde{\mathbf{w}}_\theta \in \mathbb{T}_p, \tilde{\boldsymbol{\Psi}} \text{ real symmetric}$$

$$\underset{\gamma, \gamma_l, w_{\theta_l}}{\text{minimize}} \quad \mathcal{L}_{\mathcal{G}}(\tilde{\mathbf{x}}^i; \gamma, \gamma_l, w_{\theta_l})$$
$$\gamma, \gamma_l \text{ real}, |w_{\theta_l}| = 1, \theta_1 = 0$$





iterative algorithms, sophisticated

closed forms for each parameter, simple

Content

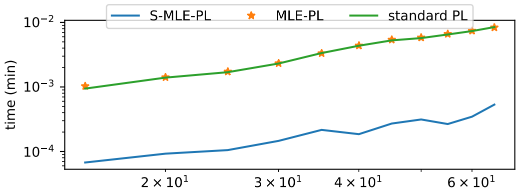
Simulation - Computation Time

Simulation parameters

- $\widetilde{\boldsymbol{\Psi}}$: Toeplitz matrix with coherence coefficient $\rho = 0.7$
- $l = p + 1 = 20$ SAR phases: random values in $[-\pi, \pi]$
- Covariance matrix : $\widetilde{\boldsymbol{\Sigma}} = \text{diag}(\widetilde{\mathbf{w}}_\theta)\widetilde{\boldsymbol{\Psi}}\text{diag}(\widetilde{\mathbf{w}}_\theta)^H$
- n i.i.d samples simulated following the $\mathcal{CN}(0, \widetilde{\boldsymbol{\Sigma}})$

S-MLE-PL	MLE-PL
$O(p^3)$	$O(n_{\text{iter}}\, p^3)$

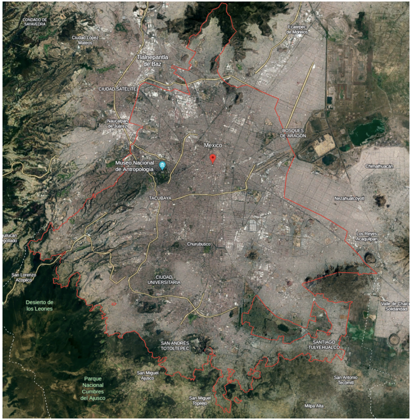
Complexity comparison of S-MLE-PL and MLE-PL



Computation time variation versus l, $n = 2 \times l$

Content

Real data - Mexico city

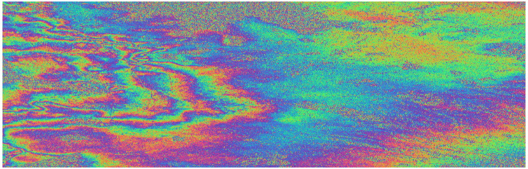


Mexico City

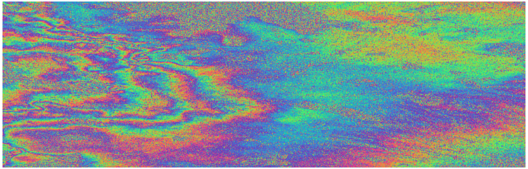
- population > 20M, highly dynamic
- rapid urbanization $\rightarrow$ increased water demand
- primary water from aquifers $\rightarrow$ subsidence and city deformation

Real data - Results

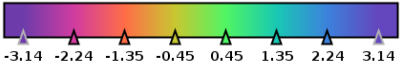
- mission: Sentinel-1
- acquisition time span: 14 August 2019 - 10 April 2020
- number of images: 20 images
- sample size: $n = 64$



MLE-PL



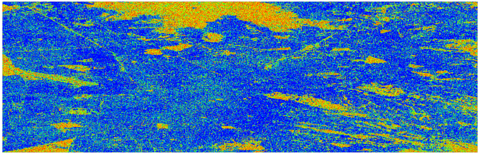
S-MLE-PL



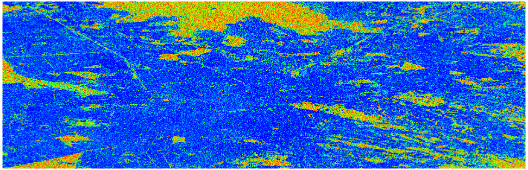
Quality assessment of phase estimation

The quality of the PL may be assessed by the goodness of the fit between the observed phases and the estimated

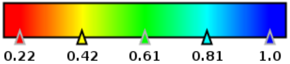
$$\gamma_{\text{post}} = \frac{\mathrm{Re}\left(\sum_{q=1}^{l} \sum_{i=q+1}^{l} e^{(\Delta\theta_{iq} - (\hat{\theta}_i - \hat{\theta}_q))}\right)}{l(l-1)/2}$$



MLE-PL



S-MLE-PL



Comparison of posteriori coherence maps estimated by MLE-PL and S-MLE-PL

Content

Conclusions

Conclusions

- **Novel approach:** efficient incorporation of new SAR images within a PL framework
- **Performance:** matches that of offline approaches (simulations as well as real data)
- **Cost:** lower computational costs than traditional offline approaches

Perspectives

- generalization of S-MLE-PL to a block of new SAR images
- estimate the displacement time series and compare the results with GPS data

[1] T. W. Anderson. *An introduction to multivariate statistical analysis*, volume 2. Wiley New York, 1958.

[2] A. Guarnieri and S. Tebaldini. On the exploitation of target statistics for sar interferometry applications. *IEEE Transactions on Geoscience and Remote Sensing*, 46(11):3436–3443, 2008.

[3] P. Vu, F. Brigui, A. Breloy, Y. Yan, and G. Ginolhac. A new phase linking algorithm for multi-temporal insar based on the maximum likelihood estimator. In *IGARSS International Geoscience and Remote Sensing Symposium*, pages 76–79. IEEE, 2022.

Thank you for your attention !

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Appendix - S-MLE-PL simulations

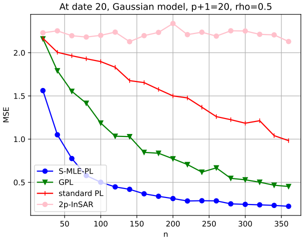
Simulation parameters

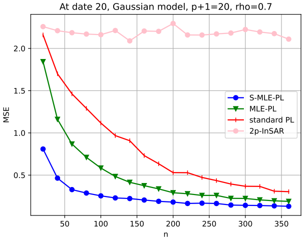
- $\widetilde{\mathbf{\Psi}}$: Toeplitz matrix with coherence coefficient $\rho \in [0.5, 0.7, 0.9]$
- $l = 20$ SAR phases: random values in $[-\pi, \pi]$
- Covariance matrix : $\widetilde{\mathbf{\Sigma}} = \text{diag}(\widetilde{\mathbf{w}}_\theta)\widetilde{\mathbf{\Psi}}\text{diag}(\widetilde{\mathbf{w}}_\theta)^H$
- n i.i.d samples simulated following the $\mathcal{CN}(0, \widetilde{\mathbf{\Sigma}})$

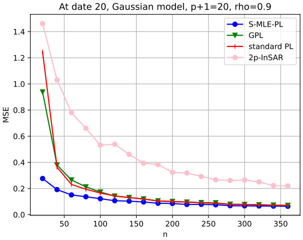
Approaches to be compared

- 2p-InSAR : phase estimated from n-pixel averaged interferograms formed with respect to the first image
- classic PL
- MLE-PL
- S-MLE-PL (our approach)

Gaussian distributed input data





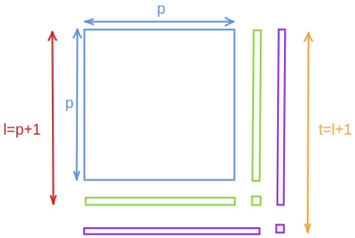


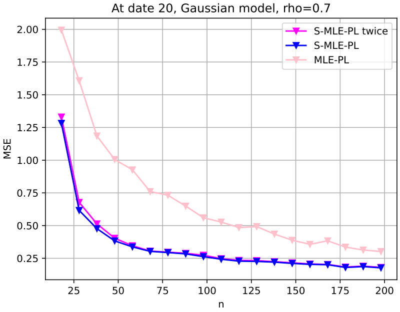
MSE of InSAR phases estimates using 2p-InSAR, classic PL and $\mathrm{MLE\text{-}PL}$ and $\mathrm{S\text{-}MLE\text{-}PL}$ with Gaussian distributed input data where $l = 20$, $\rho \in [0.5, 0.7, 0.9]$, using 1000 Monte Carlo trials

Appendix - Sequential integration of several new images

Approaches to be compared

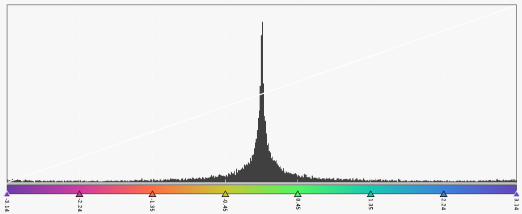
- MLE-PL processing all t images
- S-MLE-PL where the $l = p + 1$ past phases are computed using MLE-PL
- S-MLE-PL where the $l = p + 1$ past phases consist of p phases calculated using MLE-PL approach and $(p+1)^{\text{th}}$ phase calculated using S-MLE-PL

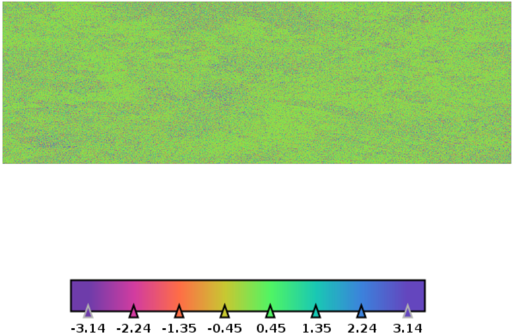




Appendix - Quality Assessment of Phase Estimation

Discrepancy between MLE-PL and S-MLE-PL





[1] H. Ansari and F. De Zan and R. Bamler, Sequential estimator: Toward efficient InSAR time series analysis, IEEE Transactions on Geoscience and Remote Sensing, 2017